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Travel of Ideas: Its Story

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Abstract

When working in a group to solve mathematical problem, students must communicate to present their idea. This idea then will be going through a series of development, refinement or rejection in argumentation and justification process. A qualitative study was conducted to explore how an idea was presented and its survival to trigger a solution to mathematical problem solving. Audio and video recording, observation and interview were conducted to collect data. From the study, it was found that an idea was presented either by speaking or combination of speaking and writing. During the development of an idea, the students communicate by speaking, listening, writing and reading to further refine the idea. This paper also discusses the challenges faced by the students in group discussion and how they handled the challenges.

Keywords: Communication, mathematics idea, mathematical problem solving, small group

1. INTRODUCTION

Mathematics learning involves activity that happens in community (Cobb, Jaworski & Presmeg, 1996; Cobb & Yackel, 1996; Forman, Larreamendy-Joerns, Steins & Brown, 1998) because development of mathematics concept involves an active social process (Hoek & Seegers, 2005). When students work in a group, they need to know the entire thinking of each member of the group (Kieran & Dreyfus, 1998). Therefore, they need to master communication skills to enable an idea to be presented, built and developed in a group. Through this collaboration, the focus changes from mental process to action because a competent student is the one who can efficiently involve in a discussion (Tatsis & Koleza, 2006). Communication can happen through speaking, reading, writing and listening (Thompson & Chappel, 2007). Through communication, mathematical ideas become the object of reflection, discussion and modification (Ministry of Education Malaysia, 2006, pp xii)

2. LITERATURE REVIEW

When working in a group, students must communicate to get information, share new idea, plan a strategy and convince others about their thinking. According to Malaysia Ministry of Education, MOE (2006), communication can happen through speaking, listening, writing and reading. When every member in a group mutually respects, appreciates ability and contribution of others, it will lead to collaborative learning (Panitz, 1996). Collaboration helps to increase students' mathematical understanding by providing them a way to critically check their reasoning and build a better new reasoning based on their friend's idea or justification (Francisco, 2013). Students depend on each other to generate, challenge, refine and produce a new idea and think as a group, and in the same way, give the needed information to solve the problem (Mueller, Maher & Yankelewitz, 2009).

According to Hoek and Seegers (2005), to ensure that the collaboration is successful, students must actively involve in collaborative problem solving, explicitly give and expand their idea, appreciate idea and contribution of others and critically evaluate every contribution. All these experiences lead to enhance thinking skills and mathematics understanding (Cross, 2009). According to Mueller, Maher and Yankelewitz (2012), there are three types of collaboration, which are: 1) co- building of justification: every member in the group mutually involves in a dialog to form a justification. 2) Integration: students' justification is strengthened by using their friends' justification. 3) Modification: students try to correct or help their friends' early unclear justification.

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Explanation and justification of mathematics idea are a part of argumentation process. Argumentation is a methodological tool that is able to explain the relationship between an individual and group; also between explanation and justification. Therefore, it has the ability to record a collective learning and relationship between an individual and group and between explanation and justification (Yackel, 2001).

Many researchers used Toulmin's Argumentation Model to study development in argumentation in a learning process (see Forman et al., 1998; Moore-Russo, Conner & Rugg, 2011; Prusak, Hershkowitz & Schwarz, 2012; Walter & Barros, 2011) because of its ability as an analysis tool to understand development of argumentation during discussion (Weber, Maher & Powell, 2008). Walter and Barros (2012) said analysis about students' argumentation provides useful information about how they reason and give meaning to problem situation. Students use problem situation to build and refine representation and understanding about unknown mathematics idea or ideas that are needed in problem solving.

Toulmin's Argumentation Model consists of three prime components namely claim (a conclusion for the argumentation), data (the source of the claim) and warrant (a justification for data to support a claim). Toulmin also introduced backing, an additional component to support warrant (Prusak, Hershkowitz & Schwarz, 2012). Prusak et al. (2012) modified Toulmin's Model by combining warrant and backing together and introducing a term reason. Figure 1(a) shows Toulmin's Schematic Argumentation and Figure 1(b) shows Prusak's simplified version.

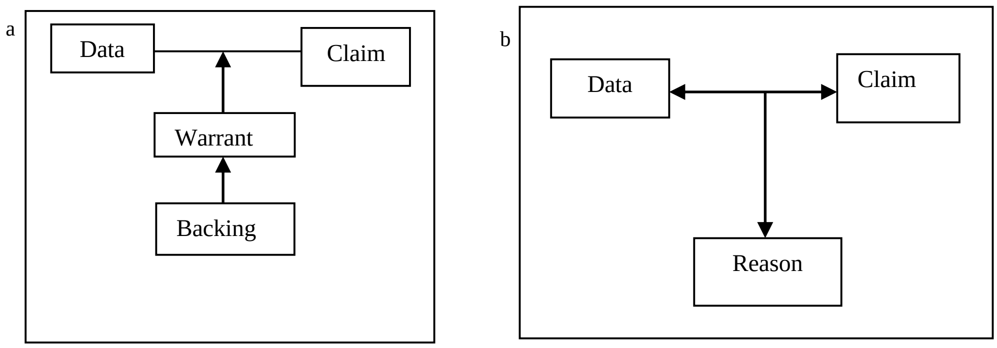


Figure 1: (a) Toulmin Schematic Argumentation; (b) A simplified Toulmin Schematic Argumentation (from Prusak et. al, 2012, pp. 27).

3. METHODOLOGY AND PARTICIPANTS

This qualitative case study aimed to explore how an idea was presented and its survival to trigger a solution to mathematical problem solving. When working in a group, students must communicate to present, build and develop mathematics' idea. Ten groups were involved in this study and each group consists of three students, aged sixteen years old, chosen by purposeful sampling. Each group was given two mathematical problems, which consist of two parts. Part (a) is a closed-typed problem whereas part (b) is an open- typed problem. The collection of data was carried out separately for each group. All group discussions were video- and audio-taped. After the discussion, each student answered the reflective questionnaire about their feelings and experiences during discussion. Then individual and group interviews were carried out. All audio and video recordings, reflective questionnaire, group and individual interviews and students' work were analysed to answer the research question. This paper will focus on Group 7, when they solved part (a) of Problem 2, that is:

> A school has 250 students who stay at hostel. In a cross-country tournament, a total of 300 students from the school, including part of the hostel students, participated in the tournament.
> The number of hostel students who did not participate in the cross-country tournament was one third of the participants who did not stay at hostel.
> Calculate the number of students who stayed at hostel but did not participate in the cross-country tournament.

Group 7 consists of one boy (Amin) and two girls (Fiqah and Syeeda). They are in the same class. Like other groups, they choose their own members in the group. According to their mathematics' teacher, all these students are good in mathematics.

4. RESEARCH FINDING

After reading the question, Amin became the first student who proposed an idea to solve the problem. He concluded that if $\frac{1}{3}$ from participants did not stay at hostel, that means another $\frac{2}{3}$ is a fraction of hostel students who participated in the cross-country. So he suggested that they multiply $\frac{2}{3}$ with 250 to get the number of students who did not stay at hostel. But when they did the multiplication, it ended up with a decimal number. So his friends rejected his idea. Fikah then suggested they used concept of probability to solve the problem. Based on Fikah's idea, they used formula $\frac{1}{3} = \frac{n(A)}{n(S)}$. By substituting n(S) with 300, they made calculation that gave n(A) as 100. But everybody did not know what was represented by the n(A) value, hence they rejected the idea.

All students reread the question. Based on the phrase "part of the hostel students participated" Syeeda said participants of the cross-country consisted of 150 hostel students and another 150 was non-hostel students. Based on the new idea, Amin concluded that hostel students who did not participate were 100. Amin and Fiqah checked the correctness of the answer whereas Syeeda seemed satisfied with her own answer. When Fiqah read the question again, she seemed unsatisfied with the answer and asked Amin for further explanation. At the same time, Amin also found the flaw in the answer.

Turn		
442	Fikah	: Wait! Wait, Amin! Given the number of hostel students who did not participate, we already know (*show on Syeeda's question sheet*), the one that we said 150, that means if we put 100 here as those who did not participate, that mean 100 is equal to $\frac{1}{3}$, right?
443	Syeeda	: Hmmm (*nod)*
444	Fikah	: Right?
445	Amin	: $\frac{1}{3}$ .. wait, it is exceeding!
446	Fikah	: The problem is, the value is not the same, it's different!!!

Fiqah checked the answer by using the equation "the number of hostel students who did not participate in cross-country tournament = $\frac{1}{3}$(the participants who did not stay at hostel)". When "the number of hostel students who did not participate" was substituted by 100, the calculation of $\frac{1}{3}$(the participants who did not stay at hostel) gave answer as 50. But calculation of (250 – 150) gave 100 as the answer. Both calculations referred to the number of hostel students who did not participate in cross-country tournament. Meanwhile Amin checked the answer by using equation $\frac{1}{3}$(non-hostel participant) = 100. It gave non-hostel participants as 300. But 300 was the total number of cross country participants. So Amin concluded that the answer was wrong because it gave the total number of cross-country participants which exceeded the given information. Figure 2 shows the schematic argumentation based on Syeeda's idea.

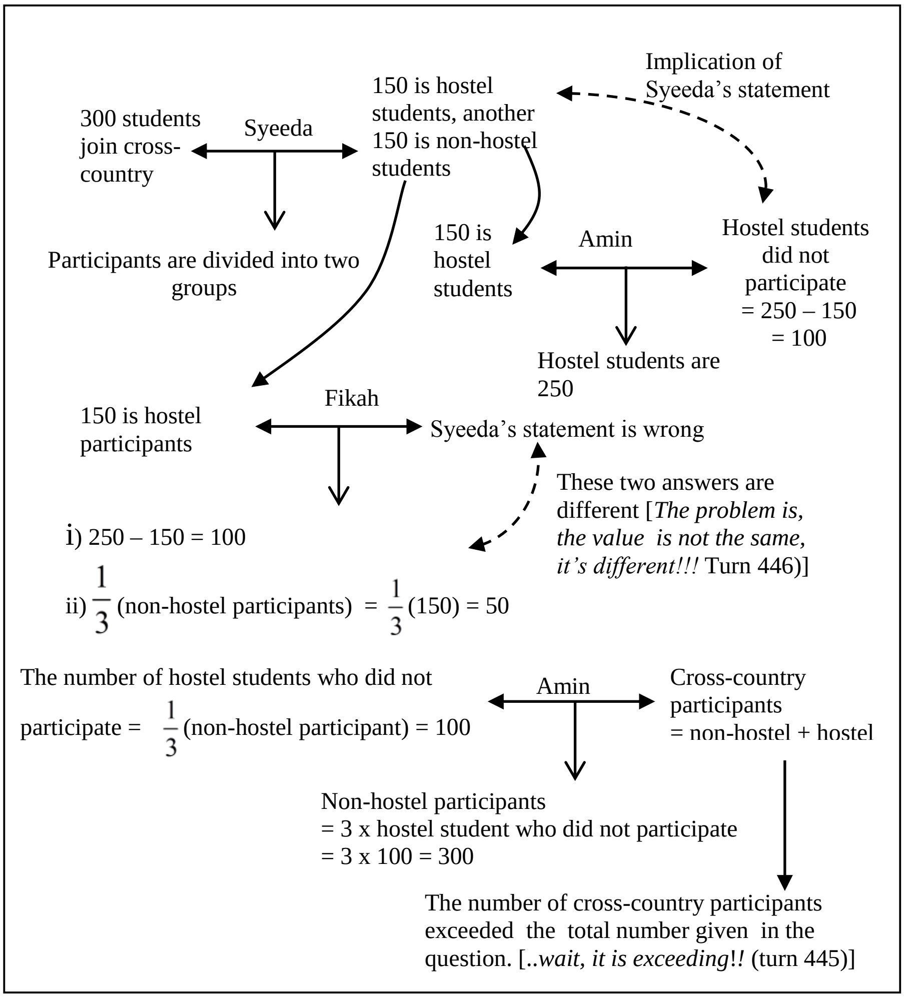


Figure 2: Schematic argumentation based on Syeeda's idea

After rejecting Syeeda's idea, everybody reread the question again. Amin then wrote 300 =? + ?. He circled both the question marks. Amin said the first question mark represented the fraction for non-hostel students and the second question mark represented the fraction for hostel students. Amin then concluded that non-hostel students were divided into three parts. But his friends disagreed with him and Amin had to make a further explanation. According to Amin, the question stated that "the number of hostel students who did not participate was $\frac{1}{3}$ from the participants who did not stay at hostel". That means, $\frac{1}{3}$ from non-hostel students participated in cross-country and another $\frac{2}{3}$ did not participate. But Syeeda insisted that Amin's idea was not logical. By using Syeeda's previous idea that non-hostel participants were 150, Amin gave further justification. According to Amin, Syeeda's statements will lead to the number of hostel students who did not participate was 50.

Turn		
605	Amin	: So the answer is 50..., hm, 50 is the answer. Try multiply 50 with 3, we get 150 here (*refer to the first question mark*),..For example, you said just now 100. So 100 multiply with 3, you get 300, here (*refer to first question mark*).
606	Syeeda	: Hm...
607a	Amin	: So it is not logical, because it is impossible nobody is here (*the second question mark*)..zero. Right?.

Figure 3 below shows a schematic argumentation for Amin justification.

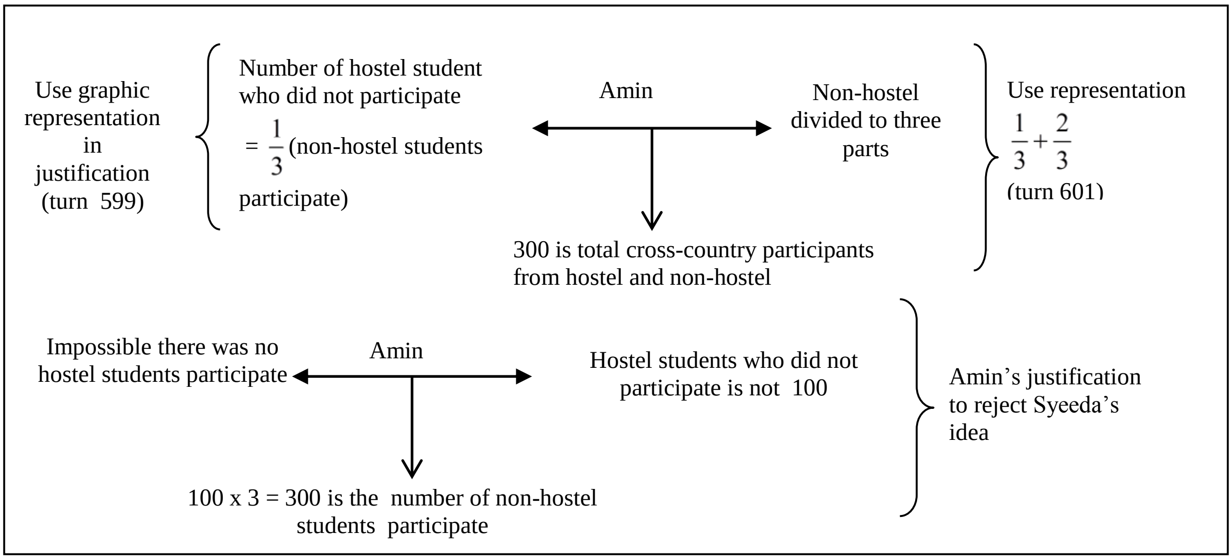


Figure 3: Schematic argumentation for Amin's justification

Amin then turned to Fiqah and explained why Fiqah's idea was wrong. According to Amin, Fiqah made mistake by jumping into conclusion that $\frac{2}{3}$ referred to the hostel participants. This conclusion was done based on Fiqah's understanding of the question.

Turn		
607b	Amin	: As for Fikah, you do like this, $\frac{1}{3}$ is non-hostel (*his finger shows to the first question mark*), you do $\frac{2}{3}$ as hostel participants (*the second question mark*). Then you divide 300 by 3 here (*shows the line for both question marks*). I divide further here , because it says ...
608	Fikah	: From...
609	Amin	: Hostel students who did not participate is $\frac{1}{3}$ from non-hostel participants (*shows the first question mark*).But you just directly divide here (*shows the value 300*). You must use this value (*refer to the first question mark*) equal to this (*refer to the phrase* '$\frac{1}{3}$ *from'*).

Figure 4 shows a schematic argumentation when Amin rejected Fiqah's idea

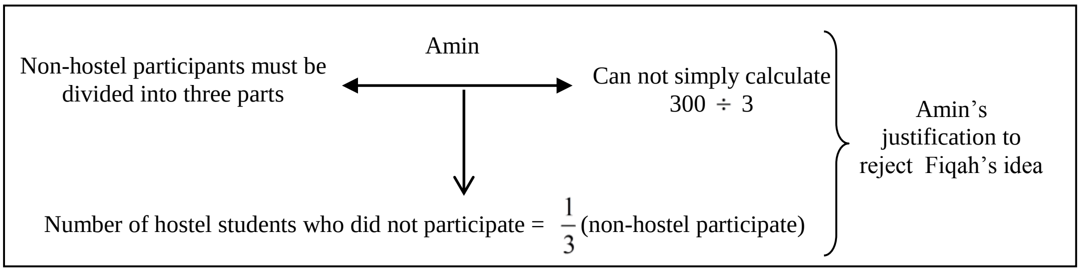


Figure 4: Schematic argumentation when Amin rejected Fiqah's idea

Amin succeeded in convincing his friends. Then they tried to solve the problems based on Amin's idea but still could not find the number of non-hostel participants. Syeeda then suggested they used linear equation. Syeeda wrote $x + y = 300$ with x represents the number of hostel students and y represents non-hostel students. Amin proposed the phrase "one third of the participants who did not stay at hostel" can be written down as $\frac{1}{3}y$ but they faced a problem because they still could not equate the term with the correct value to form an equation. Syeeda then suggested they wrote $\frac{1}{3}y = 250$. When they solved the simultaneous linear equation, they got x as -450. From the answer, Fiqah thought the second equation was still incomplete: "*Between here and here* (*refer to the formula* $\frac{1}{3}y = 250$*), either plus or minus equals to 250. It can't be just like this…!*" A schematic argumentation for the discussion is shown in Figure 5.

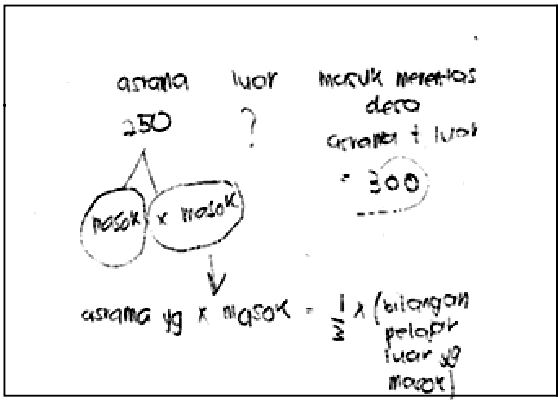
Syeeda
Amin's graphic representationk ⟷ Use simultaneous linear equation
↓
Find unknown for the number of non-hostel participant

Syeeda
Total participant = 300; Hostel students = 250 ⟷ $x + y = 300$, $\frac{1}{3}y = 250$
↓
Form two equations

Fikah
The solution of simultaneous linear equation gave $x = -450$ ⟷ " Between here and here (refer to the formula $\frac{1}{3}$ *y= 250),) either plus or minus equals to 250. It can't be just simply like this…!"* (turn 694)
↓
Equation $\frac{1}{3}y = 250$ is still incomplete

Figure 5: Schematic argumentation for group's discussion

They tried to form another equation but was still unable the find the correct equation. Amin wrote down all the given information from the question, what they would infer and what they still did not know. He said that hostel students were divided into two parts who participated and did not participate in cross-country. Then he wrote "Hostel not participate = $\frac{1}{3}$x (non-hostel participants). He also wrote the total number of cross-country participants. Figure 6 shows Amin's reperesentaion.



Hostel	Non-hostel	Participants of cross-country
250	?	Hostel and Non-hostel
Participated / Did not participate		= 300

Did not participate → Hostel non-participants = $\frac{1}{3}$ x (number of non-hostel participants)

Figure 6: Amin's representation

After listening to Amin's explanation, Syeeda concluded that if 250 is deducted from non- hostel participants, they will get the number of hostel participants. When the number is multiplied with $\frac{1}{3}$, they will get the desired answer. Syeeda explained further by saying that they can check by comparing the two calculations; that is from $\frac{1}{3}$(hostel not participate) and the answer from (250 – hostel not participate). Both of the answer must give the same value. After listening to Syeeda's explanation, Amin generated a new idea and refined his representation.

Fiqah complained they stil can not find the second equation. The complaint made Amin explained further his idea to Fiqah.

Turn
786 Amin : Look here, Fiqah. I already showed all the fractions. Hostel participant is *x*. Non-hostel participant is *y*. This *y*, $\frac{1}{3}$ from it , equal to hostel non participants (*based on his diagram*) .

Amin called his new representation as mind-mapping and it is shown in Figure 7.

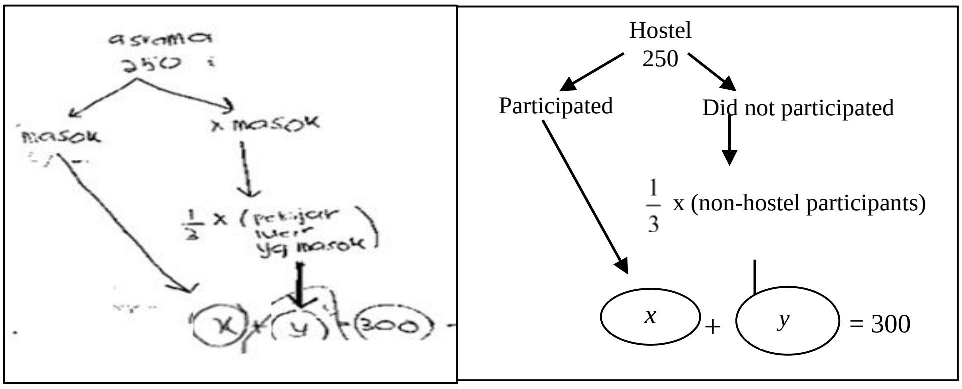


Figure 7: Amin's mind -mapping

But Amin complained they still did not know the number of hostel participants. He said he understood the question but felt that not enough information was given. Syeeda said if 150 as non-hostel participants, they will get a wrong answer. Syeeda's remarks suddenly gave idea to Amin to try trial and error method. Figure 8 shows schematics argumentation based on Syeeda's idea.

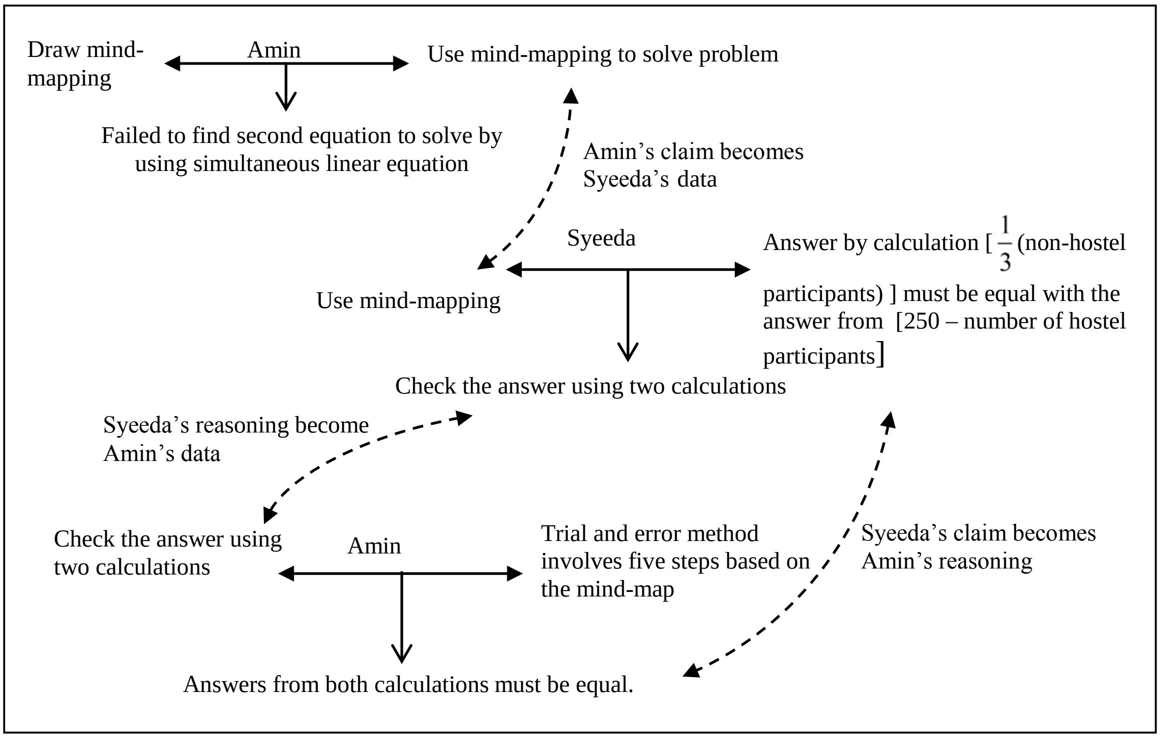


Figure 8: Schematic argumentation based on Syeeda's idea

By using his mind map, and 150 as the number of non- hostel participants, Amin showed the steps taken to check the correctness of the value. Figure 9 shows the sequences of steps taken in the trial and error method.

Turn		
801	Amin:	Hundred fifty here, divided by 3, got 50 (*wrote on his paper at value not participated*). You get 50 here (*step 2*), minus here, got 200, 200 here (write at the word participate).
802	Fikah	: Participated
803	Amin	: 200 here (*at x value*) plus hundred fifty (*y value*), got four hundred fifty. ....Not the answer. Let's try another number....

In the first step, Amin started by using 150 as the value for non-hostel participant. This value also was the value for *y*. By solving the calculation $\frac{1}{3}$ x (non-hostel participant), he got 50 as the answer. He wrote this answer in step 2. Then he minus 250 with 50 and got the answer 200 in step 3. This answer was also the value for *x* (in step 4). Lastly, by adding *x* and *y*, he got the value of 450, not 300 as desired (step 5). So, he made conclusion that the number of non-hostel participants was not 150.

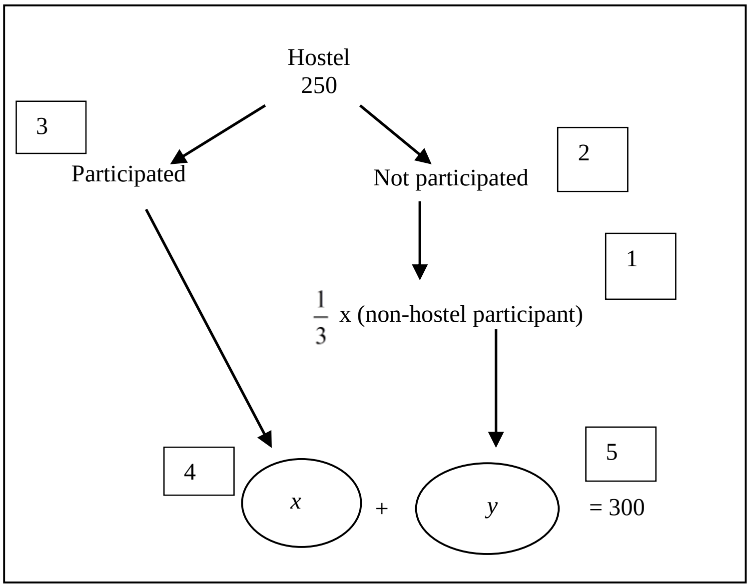


Figure 9: Sequences in trial and error method

Next, they made assumption by using *y* as 180. By using the same method, it ended up as 370, still exceeding the value given by the question. All students collaborated to find the right value of *y*. They tried using *y* as 210, 30, 240, 120 and 105. Finally, when using *y* as 25, they got the right answer.

5. DISCUSSION AND CONCLUSION

From the research, it was found that mathematics ideas travel in groups when student presents their ideas either by speaking per se or combination of speaking and writing. When they combined speaking and writing, it can happen in three ways: 1) speaking followed by writing, 2) writing followed by speaking, 3) speaking was used together with writing. For every idea that was presented, it has a chance to go through a building and development steps, although some ideas have been rejected as soon as they were presented. In building and development steps, students used combination of writing, reading, speaking and listening to communicate in a group. Research also found that students faced difficulty in reading communication. Students either failed to understand the whole problem or failed to decipher part of the phrase or sentence. Research has found that when students failed to read for understanding, it will lead to misinterpretation in the question itself. This was shown when Fiqah made a conclusion that $\frac{2}{3}$ is a fraction for non-hostel participant based on the phrase " $\frac{1}{3}$ is a fraction of hostel students who did not take part". When students fail in reading communication, it will hinder them from solving problems because mastering the language of mathematics is crucial to succeed in problem solving (Montague, Krawec & Sweeney, 2008). Meanwhile, Syeeda interprets a term "part of" as "half". Students also fail in transformation step that is to form an equation based on information given. Therefore, they cannot solve the problem using simultaneous linear equation. Weakness in reading communication also happened to another group (see Zaharah & Chew, 2015).

However, the use of listening and writing mode of communication did help students to present and develop their ideas. For example, the idea that was presented by Amin, traveled in a group, critically evaluated and challenged. He defended his idea by using a representation to convince his friends in justification process. The finding proves that the use of writing communication really did help students to generate and connect thoughts and ideas as claimed by Cross (2009). Students also used writing communication to check the correctness of idea based on their friends' idea. Meanwhile, listening to Syeeda's idea did encouraged Amin to think using his mathematical knowledge in making decisions (MOE, 2006, p xii), and helped Amin to further refine his idea to solve the problem.

The use of Toulmin's Argumentation Model did help in understanding and exploring students' learning process. From research, it was found that an idea will be rejected if the students cannot make meaning of the answer obtained or the answer was proven to be wrong. Meanwhile, an idea has a chance to go through development step if the student who presents the idea can give justification and convince his friends. Participation in collaborative discussion did help to promote learning in students when they actively request for clarification, explanation or justification (MOE, 2006). All these activities need students to have communication skill in reading, writing, listening and speaking for them to participate in group discussion.

Therefore, teachers should promote collaborative interaction among students by providing more group works. At the same time, teacher can learn from students' discussion because by finding out "why students listen, what they hear and what they do with what they heard" (Hintz, 2011,p 263) will help teacher gain a better understanding of students' experience in a mathematical discussion. Listening to students' discussion also give an opportunity to the teacher to detect students' misunderstanding and weaknesses, hence find a way to help their students. Weaknesses in reading communication must be overcome to ensure students master all aspects of communication skills that are needed in the 21st century.

However, putting students in a group and asking them to solve a problem are not enough to ensure that they discuss correctly (Mercer & Dawes, 2008). This is because students may have unclear concept about what they should do and the meaning of a good discussion (Mercer & Sams, 2006). At the same time, we are not sure whether the teachers really know exactly their role and the kind of support that should be given to their students to enhance students' collaborative activity. With the emphasis of the 21st century learning in classroom today and the need to cultivate an excellent discussion in mathematics, especially to weak students, this task might be a burden to many teachers. Therefore teachers need help and support from stakeholders for them to change their ways of teaching from teacher-centered to student-centered. The Ministry of Education can help teacher by organizing a programme to cultivate awareness among them about the importance of discussion in learning and expose them to the teaching that emphasizes exploratory discussion and argumentation in classroom. During this programme, teacher can be given training to assimilate argumentation in teaching and learning (Prusak, 2002), hence become more effective in teaching because they have an in-depth understanding about a good discussion (Lawson, 2002 in Qunn & Wing, 2012). Simultaneously, examination should be more on open question to encourage student to communicate and give reasoning in their answers. We believe that this kind of assessment will push teachers and students to be more critical in their mathematical thinking, hence more actively participate in exploratory discussion.

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